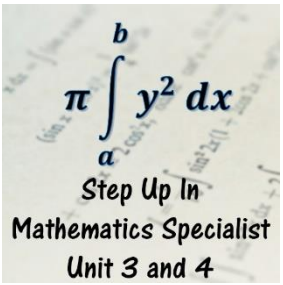
1.3 The Complex Plane

Problems Worksheet



1. For the complex numbers z and w

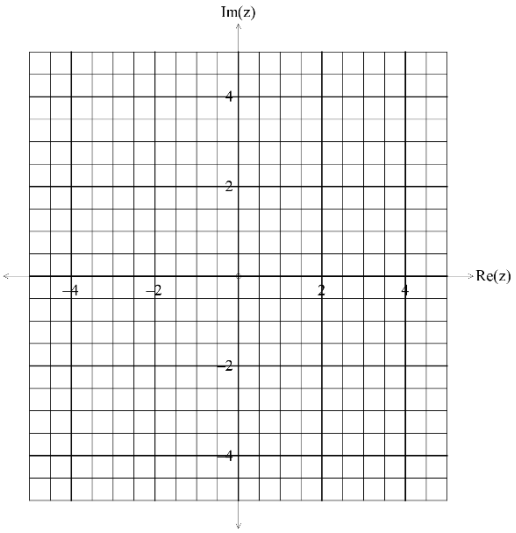
   a. Show that $z\overline{w} + w\bar{z} = 2Re(z\overline{w})$

   b. Hence show that $|z + w| \leq |z| + |w|$

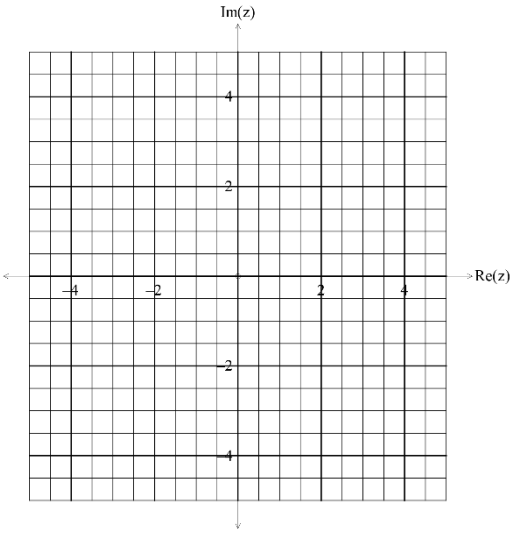
   c. In your own words, state the physical meaning of the inequality in (b).

2. Sketch the locus of z such that:

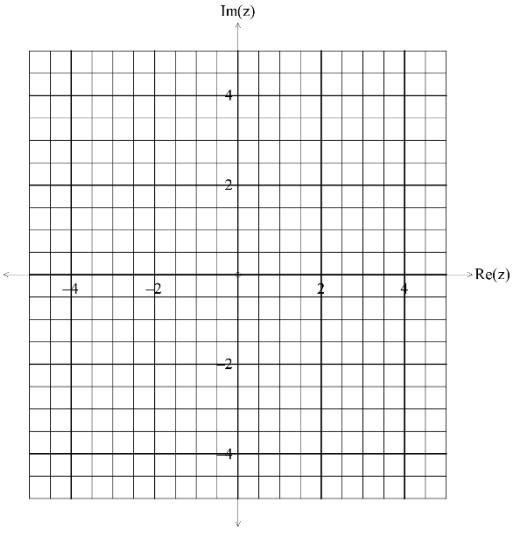
a. $z\bar{z} = 3$



b. $z + \bar{z} = 8$

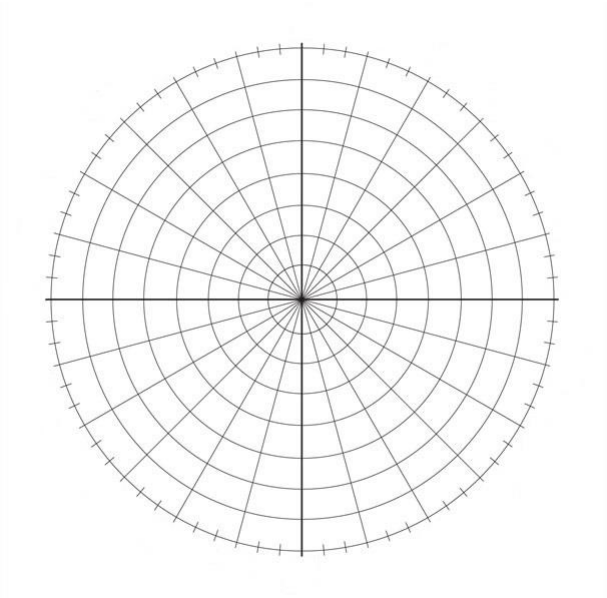


c. $arg(z) + arg(\sqrt{3} - i) = \frac{\pi}{3}$

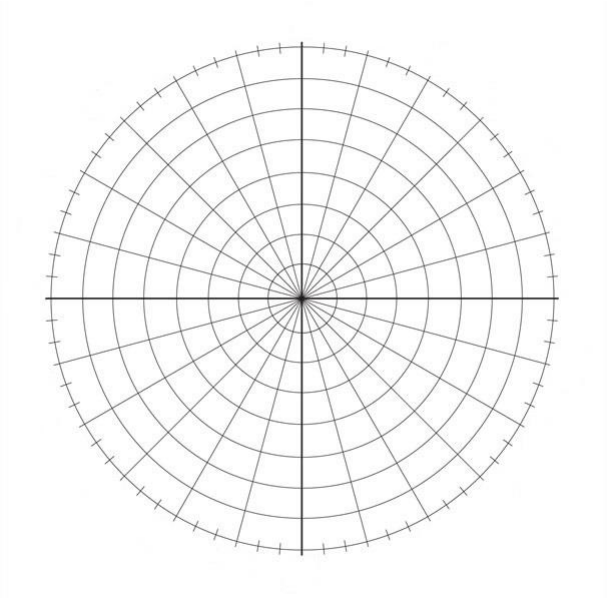


3. Let $J = \left\{z: z = \sqrt{2}\,\text{cis}\,\theta\,, 0 \leq \theta \leq \frac{\pi}{3}\right\}$. Sketch:

a. J

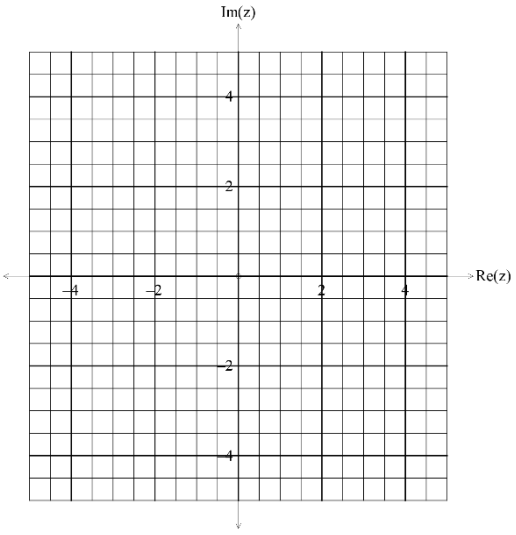


b. $K = \{w: w = z^2\}$

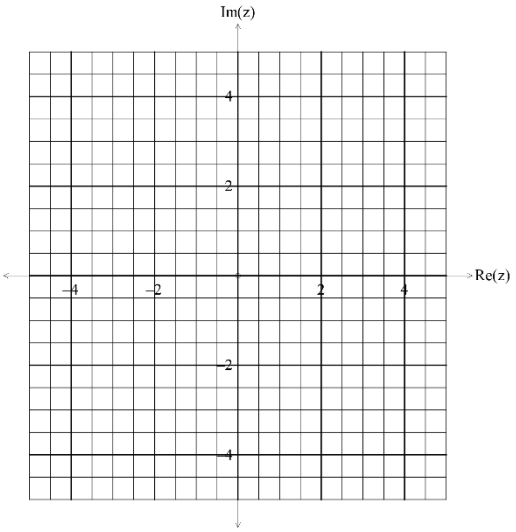


4. Sketch the locus of z such that:

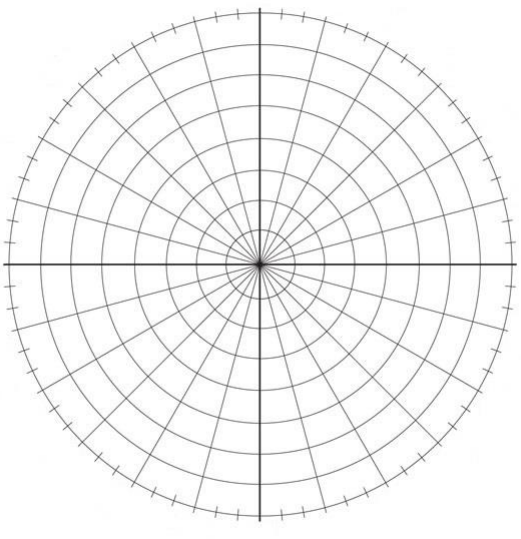
   a. $Re(z) \leq 2Im(z) + 1$



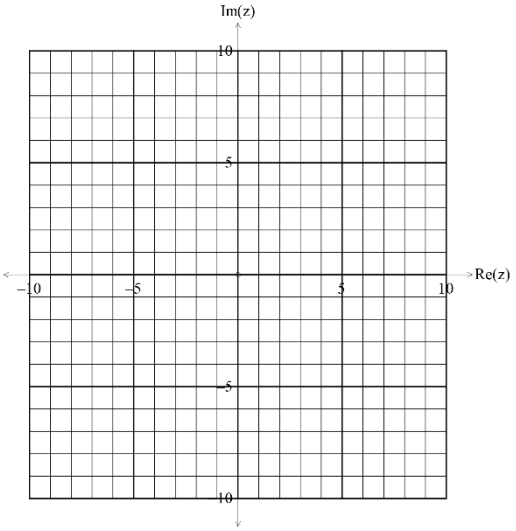
   b. $\left\{z: 2 \leq |z| < 4 \cap arg(z) \geq \frac{3\pi}{4}\right\}$



   c. $arg(iz) = \frac{\pi}{3}$

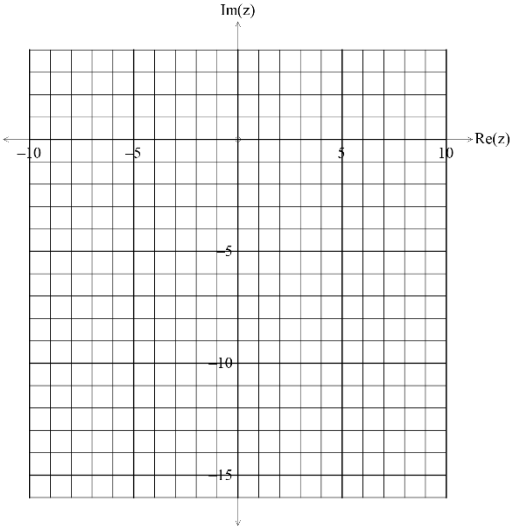


5. Sketch the locus of z defined by $\{z: |z + 3 - 2i| + |z + 3 - 6i| = 8\}$.



6. Sketch the locus of z given that:

   a. $Im(z) = |z - 2 + i|$



   b. $Re(z) = |z + 3 + 3i|$